

**초탄성 비선형 재료 모델을 포함하는
Partition of Unity 기반 사면체 유한요소 개발**

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2024.12.06



Committee

- Prof. Eunho Kim, Chair (Mechanical System Engineering, Jeonbuk National University)
- Prof. Donghwi Lee (Mechanical System Engineering, Jeonbuk National University)
- Prof. Hyungmin Jun (Mechanical System Engineering, Jeonbuk National University)

Goal

Resolved the volumetric locking issue of the basic tetrahedral elements used for the structural analysis of hyperelastic material models and enhanced the accuracy of the numerical analysis

Introduction

- Characteristics and Advantages of Hyperelastic Materials
- Numerical Modeling of Hyperelastic Materials

Partition of Unity-based Finite Elements

- High-Order Approximations in Partition of Unity-Based Finite Element

Nonlinear Static Analysis of Partition of Unity-based Finite Element

- Procedure for Nonlinear Static Analysis of Partition of Unity-Based Finite Element
- Modeling of Hyperelastic Materials & Large Deformation Kinematics
- Numerical Example of Nonlinear Static Analysis

Nonlinear Modal Analysis of Partition of Unity-based Finite Element

- Procedure for Nonlinear Modal Analysis
- Numerical Example of Nonlinear Modal Analysis

Future Works

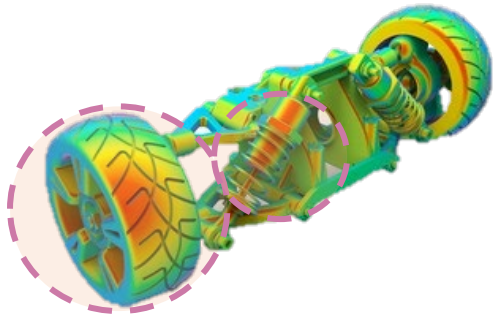
Conclusions

What is a Hyperelastic Material?

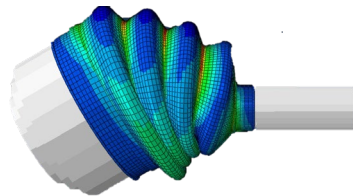
Hyperelastic materials return to their original state when the external load is removed and exhibit nonlinear stress-strain behavior.

Applications of Hyperelastic Materials

Designing Mechanical Components

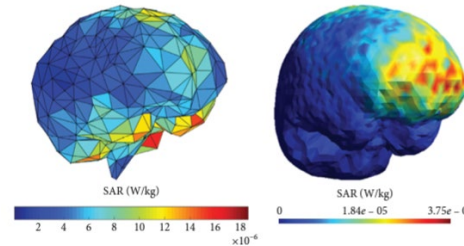


Automotive Suspension Systems



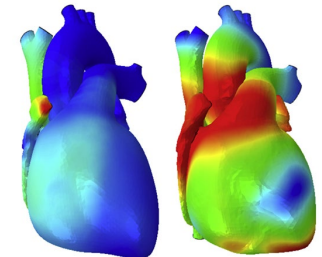
Rubber Boot Seals

Simulation of Biological Tissues



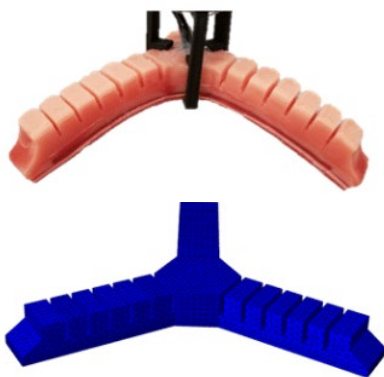
Cvetković, Dodig, and Poljak., Math. Probl. Eng, 1-18, 2020

Virtual Brain Model

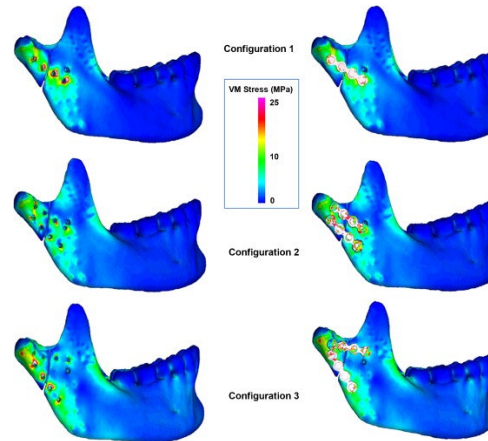


Baillargeon et al., Eur.J.Mech, 48, 38-47, 2014

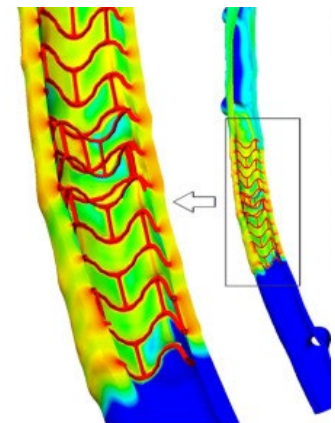
Virtual Cardiac Model



Soft robotics



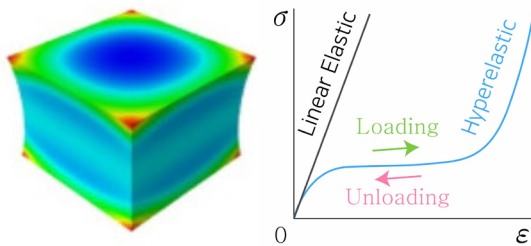
The Mandibular Condyle



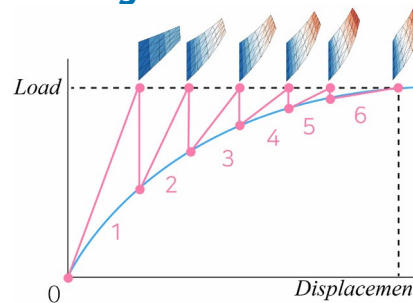
Coronary Artery Stent

Challenges in Numerical Modeling of Hyperelastic Structures

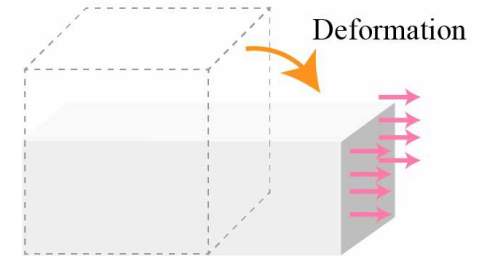
Material Nonlinearity



Large Deformation



Incompressibility

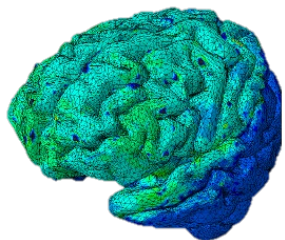


- **Nonlinear Relation:** Nonlinear stiffness-displacement relation causing convergence issues
- **Volumetric Locking:** Excessively stiff response due to the failure to satisfy incompressibility

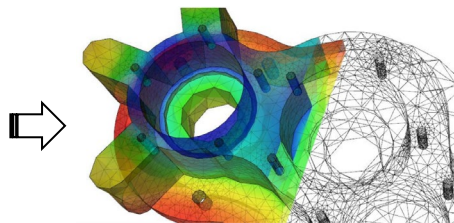
Pagani, J., et al., Mech. Adv. Mater. Struct, 31, 117-37, 2024

Four-node Tetrahedral Finite Element

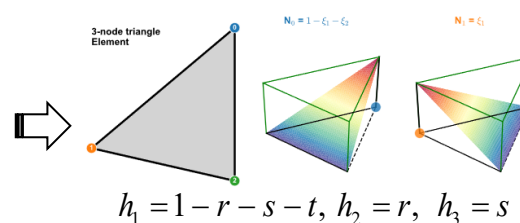
Complex Structure



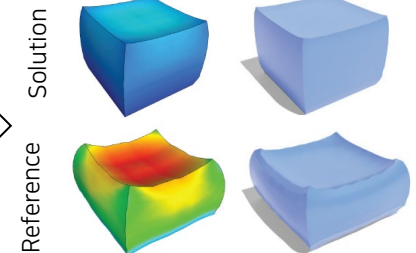
Tetrahedral Element



Linear Interpolation



Volumetric Locking



Reduction in Solution Accuracy and Design/Evaluation Reliability

Displacement Interpolation

Standard FEM $\mathbf{u}(r, s, t) = \sum_{i=1}^4 h_i(r, s, t) u_i$ ①

PU-based FEM $\mathbf{u}(r, s, t) = \sum_{i=1}^4 h_i(r, s, t) \bar{\mathbf{u}}_i(\mathbf{x})$ ②
with $\bar{\mathbf{u}}_i(\mathbf{x}) = [\bar{u}_i, \bar{v}_i, \bar{w}_i]^T$

$\hat{h}_i(r, s, t)$: Shape Function
 u_i : Nodal Displacement Variable
 $\bar{\mathbf{u}}_i(\mathbf{x})$: Local Approximation Function

Local Approximation, $\bar{\mathbf{u}}_i(\mathbf{x})$

$\bar{u}_i = \mathbf{p}_i(\mathbf{x}) \mathbf{a}_i^u$ $\bar{v}_i = \mathbf{p}_i(\mathbf{x}) \mathbf{a}_i^v$ $\bar{w}_i = \mathbf{p}_i(\mathbf{x}) \mathbf{a}_i^w$ ③

$\mathbf{p}_i(\mathbf{x}) = [1, x_i, y_i, z_i, x_i^2, xy_i, xz_i, y_i^2, yz_i, z_i^2 \dots]$ ④

$\mathbf{a}_i^u = [a_{1i}^u, a_{2i}^u, a_{3i}^u, a_{4i}^u]^T$ ⑤

$\mathbf{p}_i(\mathbf{x}) \mathbf{a}_i^u = a_{1i}^u + a_{2i}^u x + a_{3i}^u y + a_{4i}^u z$ ⑥

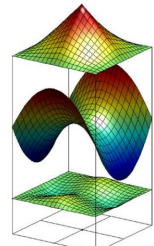
$u_i(x) = a_{1i}^u + a_{2i}^u x + a_{3i}^u y + a_{4i}^u z = \bar{u}_i$ ⑦

$a_{1i}^u = \bar{u}_i - a_{2i}^u x_i - a_{3i}^u y_i - a_{4i}^u z_i$ ⑧

$\mathbf{p}_i(\mathbf{x})$: Polynomial basis vector

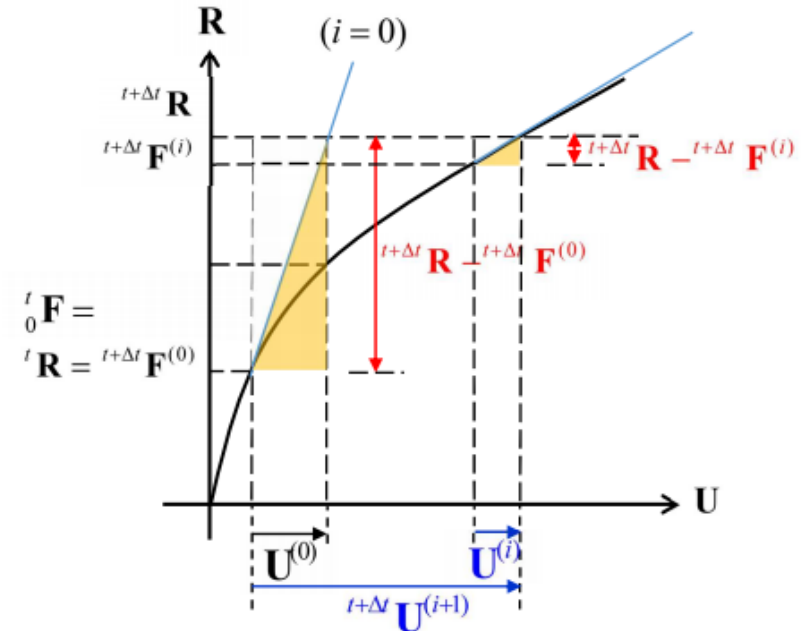
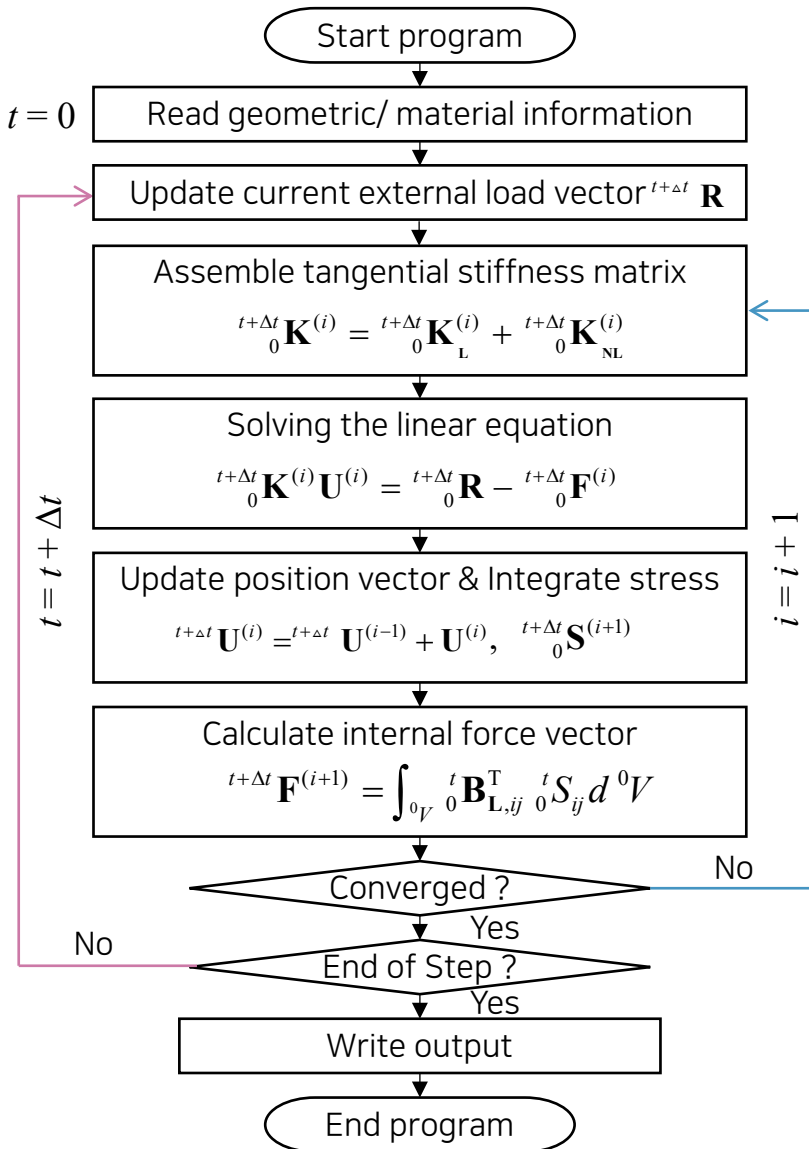
$\mathbf{a}_i^u, \mathbf{a}_i^v, \mathbf{a}_i^w$: Polynomial basis coefficients

$$\mathbf{u}(r, s, t) = \sum_{i=1}^4 h_i(r, s, t) \begin{bmatrix} \bar{u}_i + a_{2i}^u(x - x_i) + a_{3i}^u(y - y_i) + a_{4i}^u(z - z_i) \\ \bar{v}_i + a_{2i}^v(x - x_i) + a_{3i}^v(y - y_i) + a_{4i}^v(z - z_i) \\ \bar{w}_i + a_{2i}^w(x - x_i) + a_{3i}^w(y - y_i) + a_{4i}^w(z - z_i) \end{bmatrix}$$

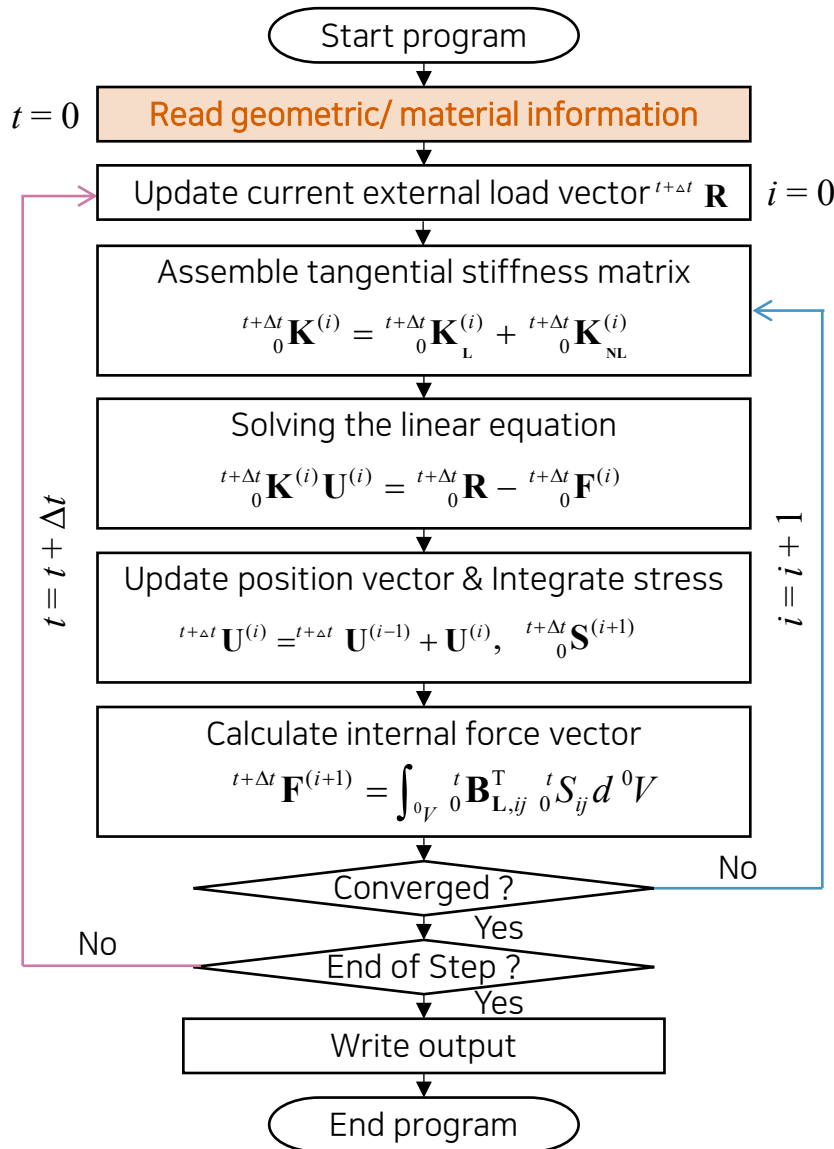
$$\mathbf{u}(r, s, t) = \underbrace{\sum_{i=1}^4 h_i(r, s, t) \bar{\mathbf{u}}_i}_{\text{Standard Term}} + \underbrace{\sum_{i=1}^4 \hat{h}_i(r, s, t) \hat{\mathbf{u}}_i}_{\text{Standard Term}}$$


- ➡ Prevent volumetric locking, ensuring accurate deformation for incompressible materials
- ➡ Improving the accuracy of solutions, ensuring convergence in nonlinear analyses

FE Solution Procedures

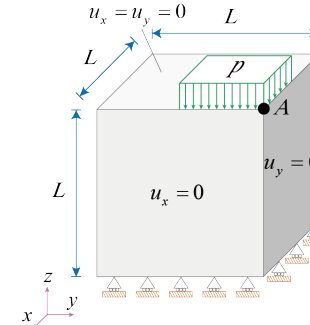


FE Solution Procedures



Read Geometric/ Material Information

Geometric/Material Information



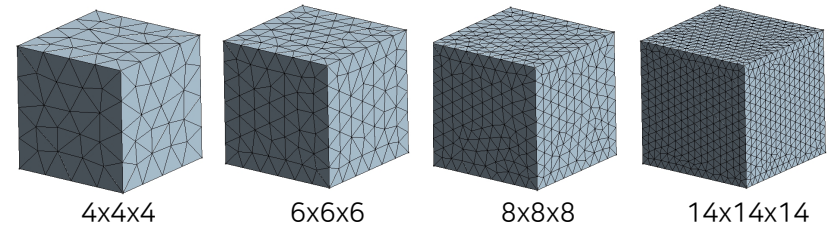
Neo-Hookean Material

• $A_{10} = 0.5 \text{ Pa}$, $k = 100 \text{ Pa}$

Length & Pressure

• $L = 10 \text{ cm}$, $p = -0.5 \text{ N/m}^2$

Mesh Refinement Levels



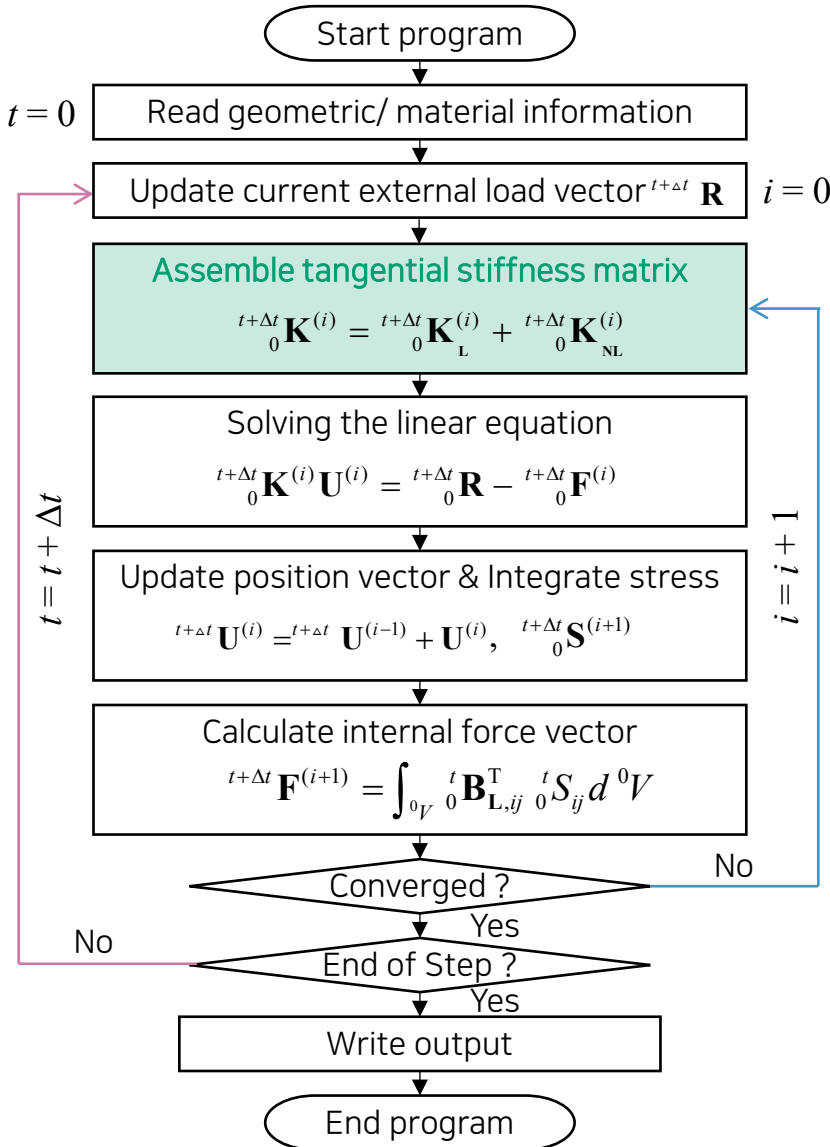
Increasing the number of elements improves accuracy

Data Structure for Element Representation

Data Structure	
Problem Dimension	3
# of Nodes per Element	4
# of DOFs per Node	3+ 9(Enriched DOFs)
# of Gauss Point	11

Resolving the linear dependency issue by constraining all DOFs at the boundaries

FE Solution Procedures



Assemble Tangential Stiffness Matrix

Hyperelastic Material Model

- Mooney-Rivlin and Neo-Hookean material model

- Strain density function

$$W(J_1, J_2, J_3) = A_{10}(J_1 - 3) + A_{01}(J_2 - 3) + \frac{K}{2}(J_3 - 1)^2$$

- Second Piola-Kirchhoff stress tensor

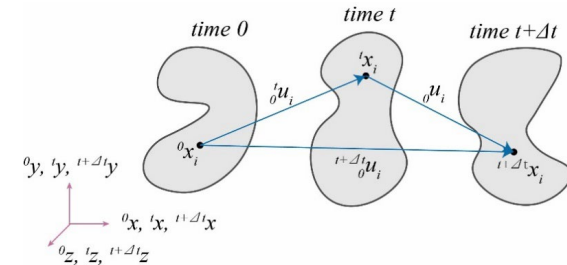
$${}^tS_{ij} = A_{10} \frac{\partial {}^tJ_1}{\partial {}^t\varepsilon_{ij}} + A_{01} \frac{\partial {}^tJ_2}{\partial {}^t\varepsilon_{ij}} + \kappa \frac{\partial ({}^tJ_3 - 1)}{\partial {}^t\varepsilon_{ij}}$$

- Fourth-order constitutive tensor

$${}^tD_{ijrs} = A_{10} \frac{\partial^2 {}^tJ_1}{\partial {}^t\varepsilon_{ij} \partial {}^t\varepsilon_{rs}} + A_{01} \frac{\partial^2 {}^tJ_2}{\partial {}^t\varepsilon_{ij} \partial {}^t\varepsilon_{rs}} + \kappa \frac{\partial ({}^tJ_3 - 1)}{\partial {}^t\varepsilon_{ij}} \frac{\partial {}^tJ_3}{\partial {}^t\varepsilon_{rs}} + \kappa \frac{\partial {}^tJ_3}{\partial {}^t\varepsilon_{ij}} \otimes \frac{\partial {}^tJ_3}{\partial {}^t\varepsilon_{rs}}$$

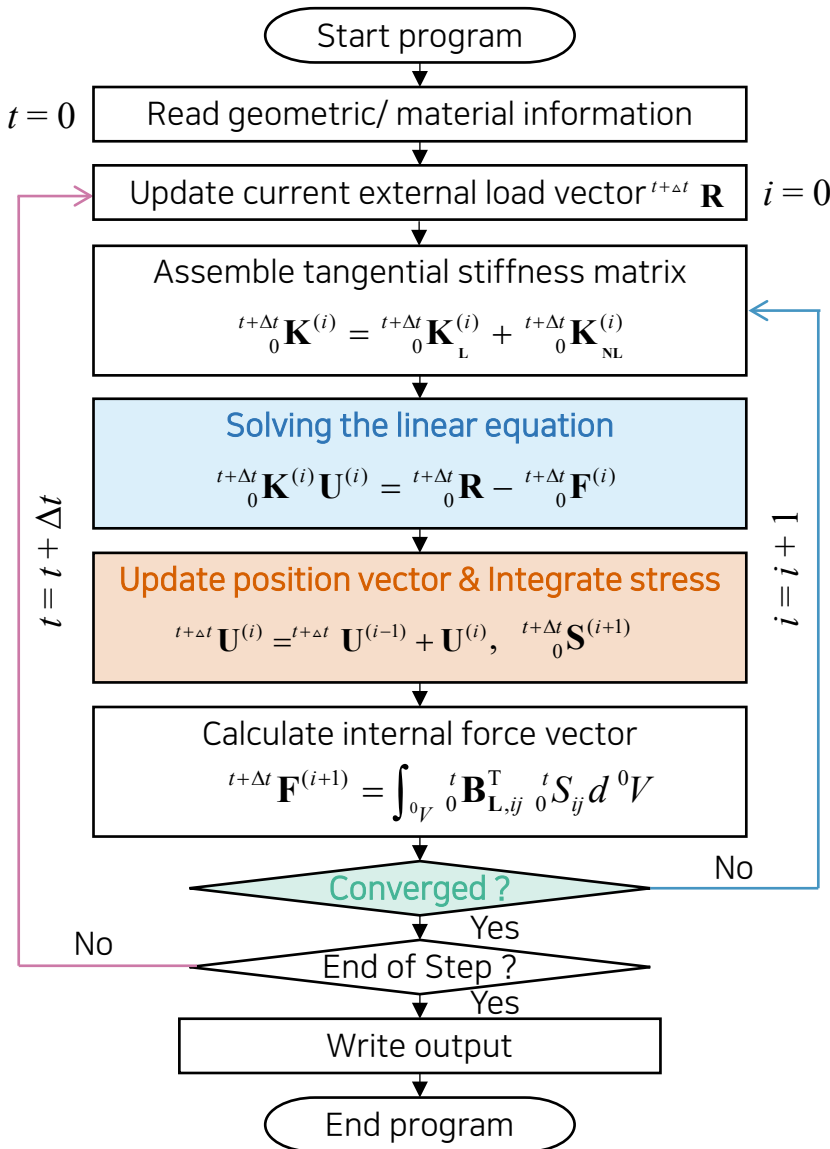
Large Displacement Kinematics Formulation

- Total Lagrangian formulation



$${}^t\mathbf{K}_L = \int_{0V} {}^t\mathbf{B}_{ij}^T {}^tD_{ijkl} {}^t\mathbf{B}_{kl} d^0V \quad {}^t\mathbf{K}_{NL} = \int_{0V} {}^t\mathbf{N}_{ij}^T {}^tS_{ij} {}^t\mathbf{N}_{ij} d^0V$$

FE Solution Procedures

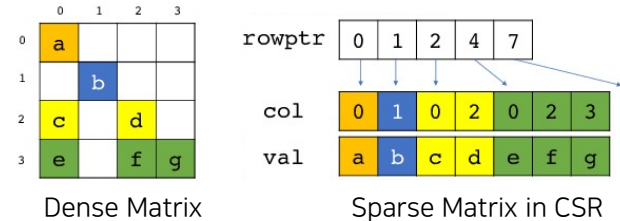


Solving the Linear Equation

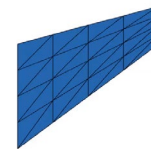
Pardiso Solver

$$\mathbf{U}^{(i)} = {}^{t+\Delta t}_0\mathbf{K}^{(i-1)-1} \left[{}^{t+\Delta t}_0\mathbf{R} - {}^{t+\Delta t}_0\mathbf{F}^{(i)} \right]$$

- High-performance linear equation solver included in Intel's Math Kernel Library (MKL)
- Optimized memory efficiency using the Compressed Sparse Rows (CSR) format

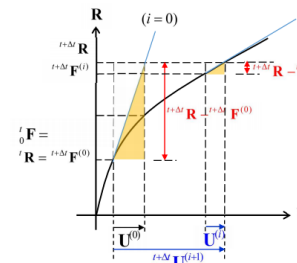


❖ Update Position Vector



- Separates displacement DOFs and additional DOFs for iterative updates

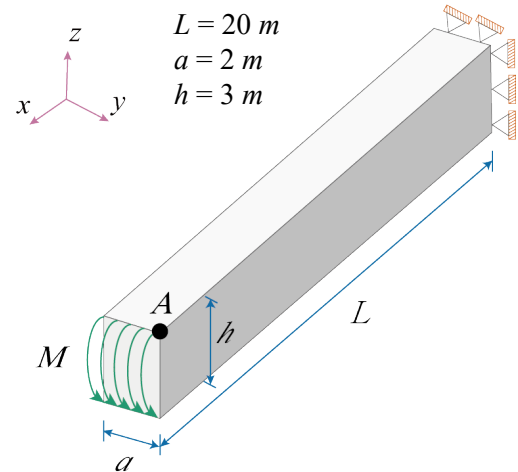
❖ Energy-based Convergence Check



- Calculated by minimizing the difference between internal force $\mathbf{F}$ and external force $\mathbf{R}$
- Allowable edge error $e_E = 0.0001$

Finite Element Capabilities for Volumetric Locking under Poisson's Ratio Variation

Problem Description



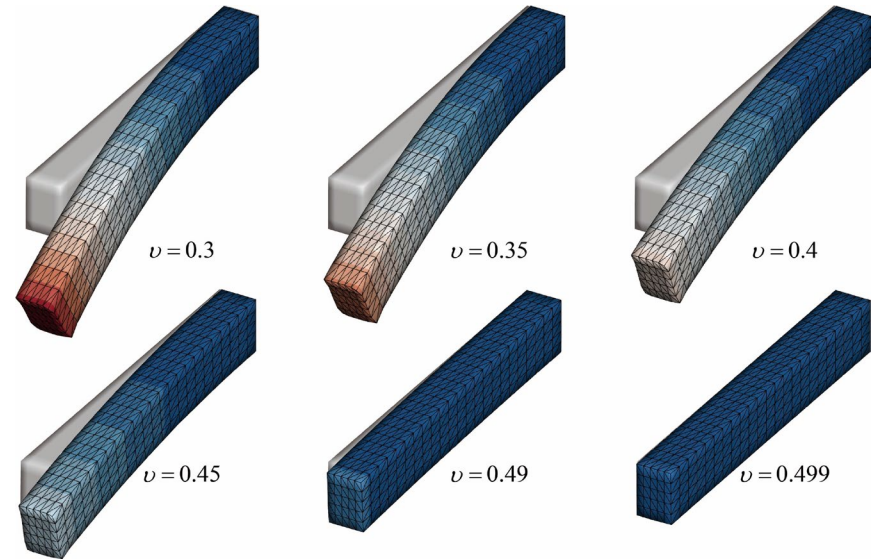
Material Property

- Elastic Material
- $E: 300 \text{ GPa}$
- Poisson Ratio: $0.3 \sim 0.499$

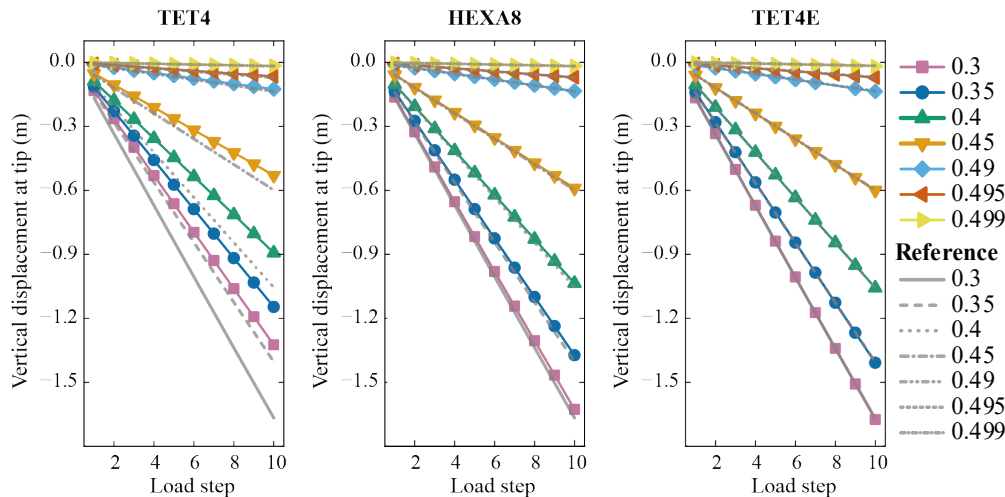
Problem Property

- Dimension: 3
- $M = 10 \text{ Nm}$
- Step: 20
- Tolerance: 0.001

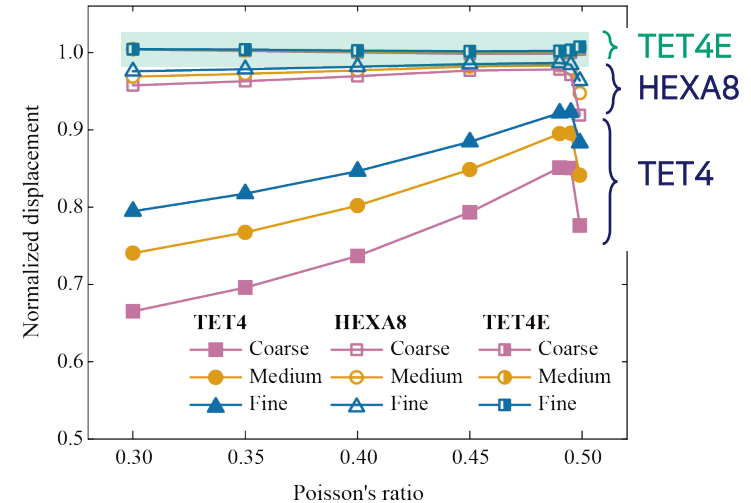
Deformed Shapes of Cantilever Beam with TET4E



Calculated Load-displacement Curves at Point A

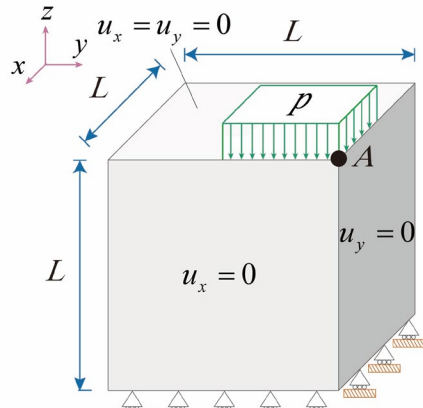


Comparison of Normalized Displacement



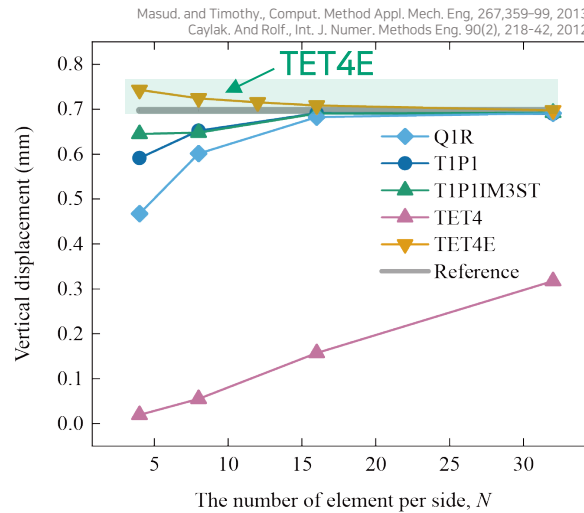
Nearly Incompressible Block under Compression

Problem Description

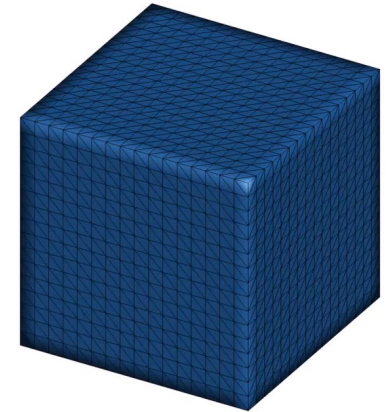


- $A_{10} = 40.097 \text{ MPa}$
- $L = 10 \text{ mm}$
- $k = 400,889.806 \text{ MPa}$
- $p = 320 \text{ Mpa}$

Convergence Curve



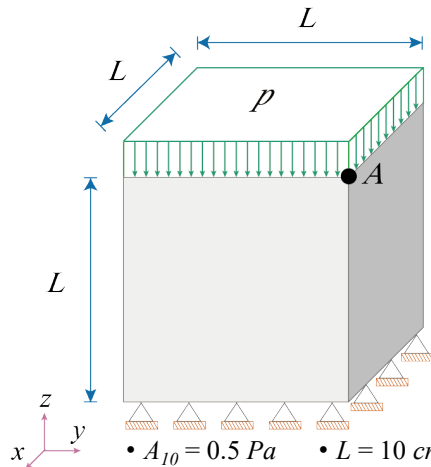
Deformed Shape of TET4E



TET4E

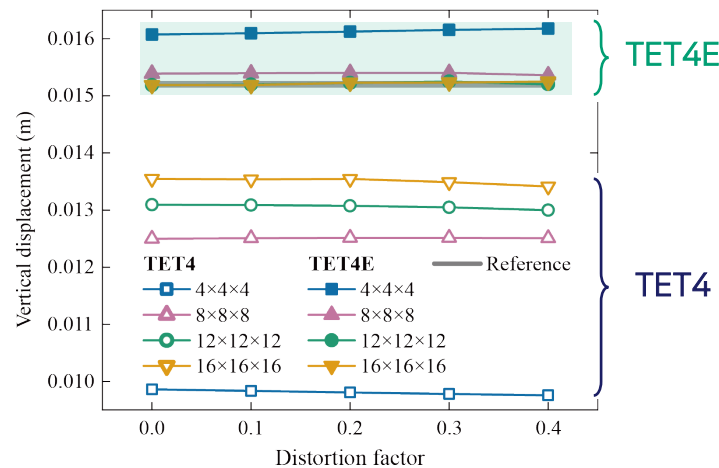
Mesh Distortion Test

Problem Description

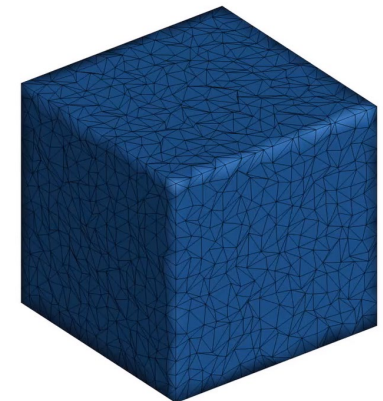


- $A_{10} = 0.5 \text{ Pa}$
- $L = 10 \text{ cm}$
- $k = 100 \text{ Pa}$
- $p = -0.5 \text{ N/m}^2$

Displacement with Distortion Change



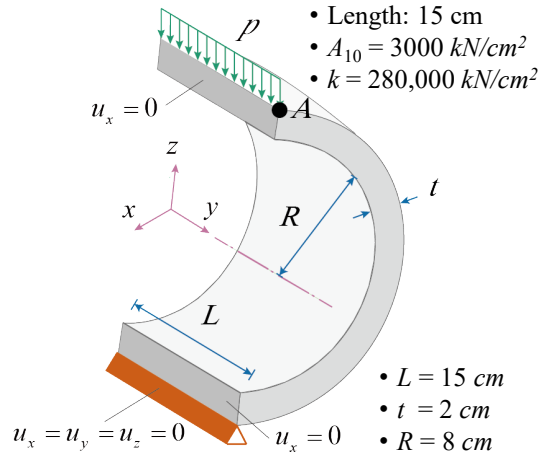
Deformed Shape of TET4E



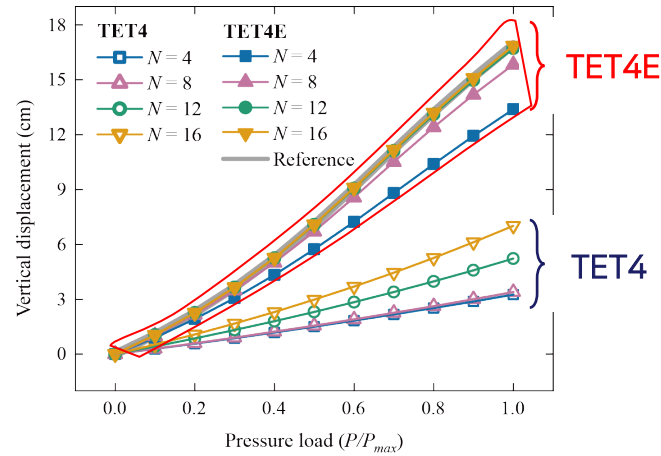
TET4E

Nearly Incompressible Cylinder under Line Load

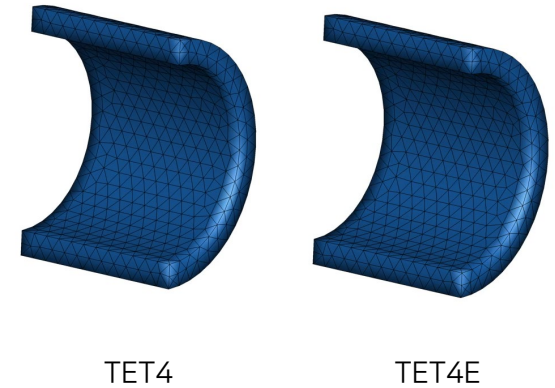
Problem Description



Load-Displacement Curve

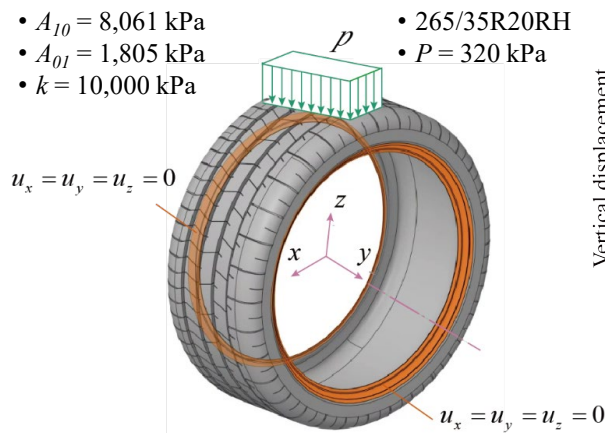


Deformed Shape

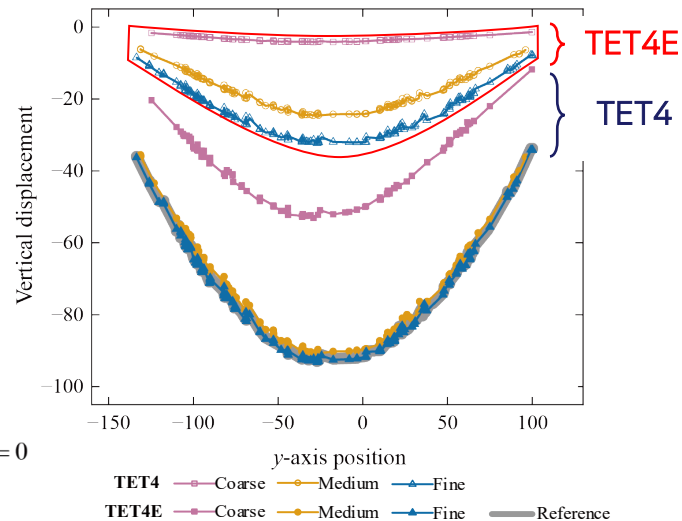


Hyperelastic Tire under Compression

Problem Description



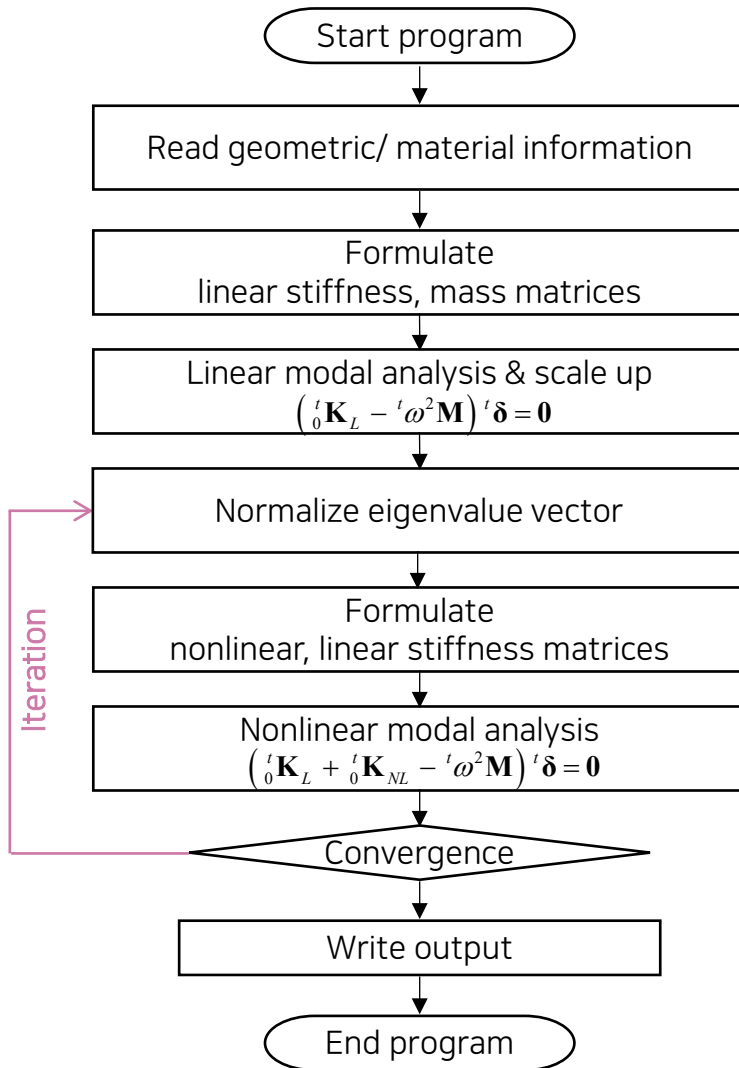
Vertical Displacement Curve



Deformed Shape

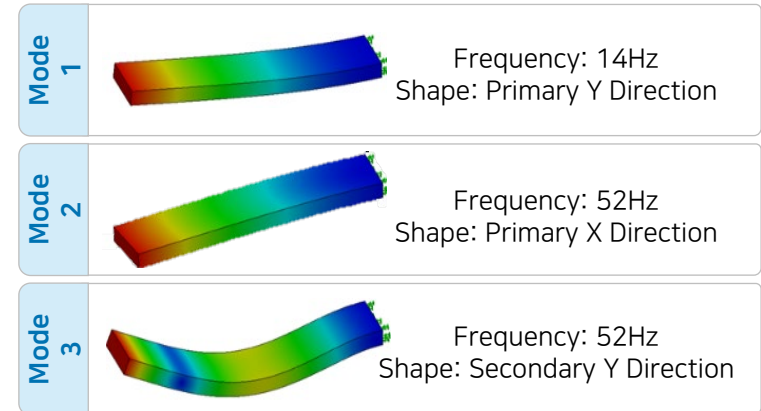


Nonlinear Modal Analysis Procedure



Modal Analysis

- Modal analysis is the study of the dynamic properties of systems in the frequency domain



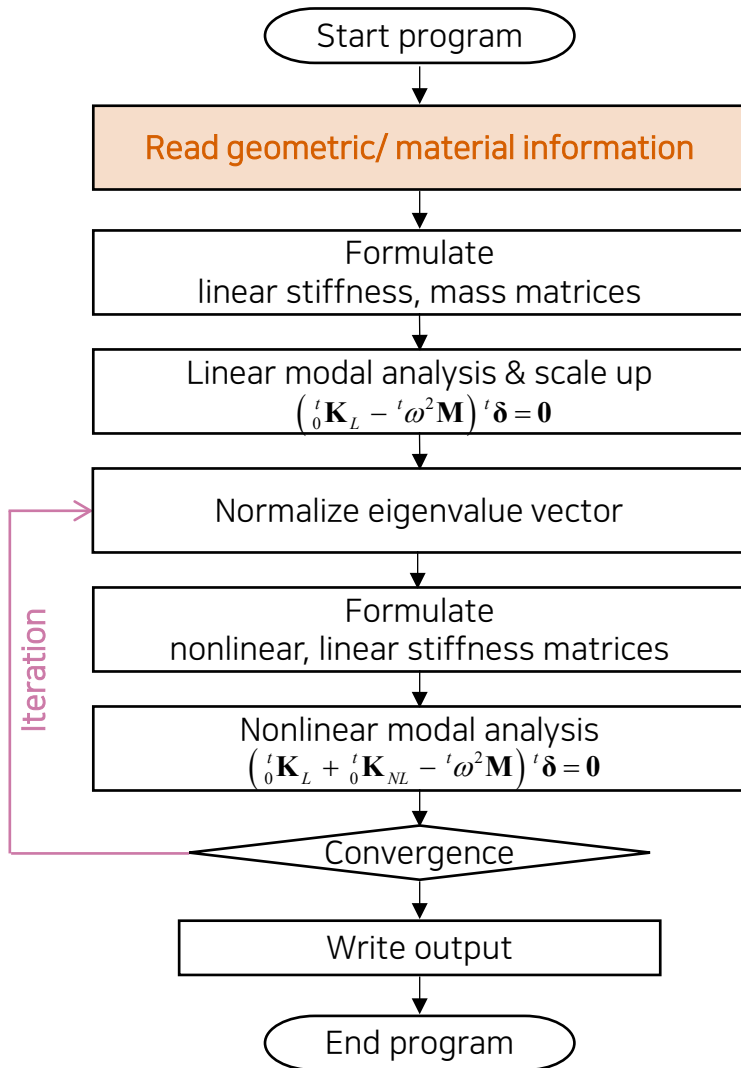
Amplitude-based Nonlinear Modal Analysis

- The nonlinearity of a hyperelastic material has a different response depending on the deform
- Linear mode analysis does not reflect these nonlinear characteristics



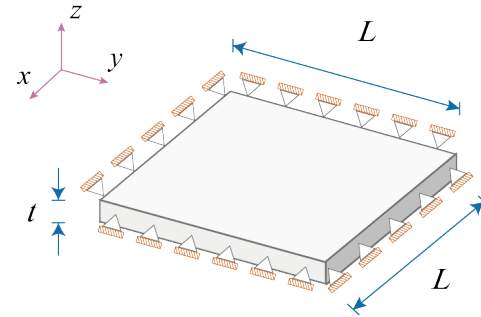
Predict the exact behavior of the system with nonlinear mode analysis

Nonlinear Modal Analysis Procedure



Read Geometric/ Material Information

Geometric/Material Information



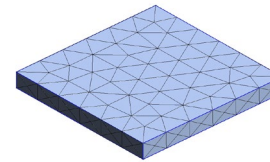
Mooney-Rivlin Material

- $A_{10} = 100 \text{ kN/m}^2$
- $A_{01} = 50 \text{ kN/m}^2$
- $k = 2,000 \text{ kN/m}^2$
- $\rho = 1,000 \text{ kg/m}^3$

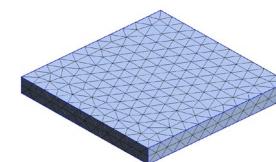
Length & Pressure

- $L = 10 \text{ cm}$, $t = 1 \text{ cm}$

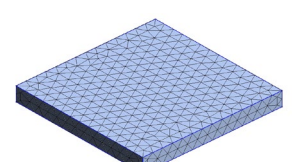
Mesh Refinement Levels



Coarse mesh



Medium mesh



Fine mesh

Increasing the number of elements improves accuracy

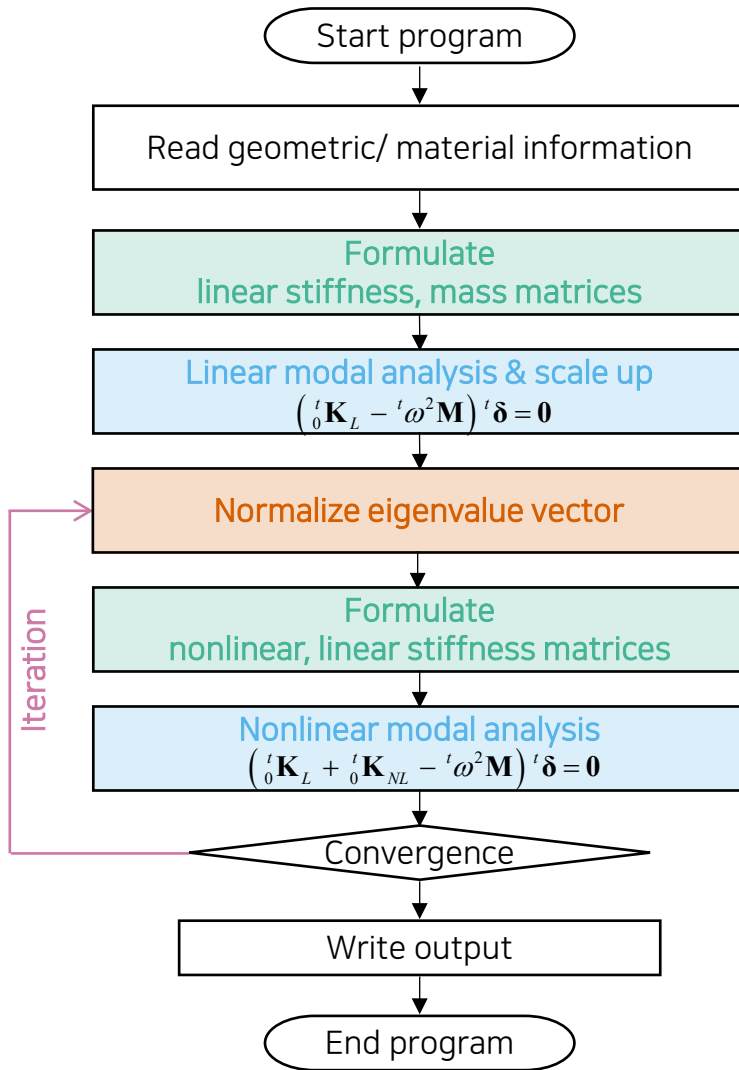
Data Structure for Element Representation

Data Structure	
Problem Dimension	3
# of Nodes per Element	4
# of DOFs per Node	3+ 9(Enriched DOFs)
# of Gauss Point	11



Resolving the linear dependency issue by constraining all DOFs at the boundaries

Nonlinear Modal Analysis Procedure



Formulate Stiffness & Mass Matrices

Consistent Mass Matrix

- Unlike the Lumped Mass Matrix, the density is uniformly distributed within the element.

$$\mathbf{M} = \int_{0V} h^T \rho h dV.$$

ρ : Material mass density
 h : Shape function matrix
 V : Element volume

Linear and Nonlinear Stiffness Matrices

- Stiffness matrix is improved using PU-based approximation, enhancing convergence and accuracy in nonlinear modal analysis

$${}^t\mathbf{K}_L = \int_{0V} {}^t\mathbf{B}_{ij}^T D_{ijkl} {}^t\mathbf{B}_{kl} d^0V \quad {}^t\mathbf{K}_{NL} = \int_{0V} {}^t\mathbf{N}_{ij}^T S_{ij} {}^t\mathbf{N}_{ij} d^0V$$

Normalize Eigenvalue Vector

- Linear mode shapes are scaled and normalized to match the desired amplitude ratio at maximum displacement

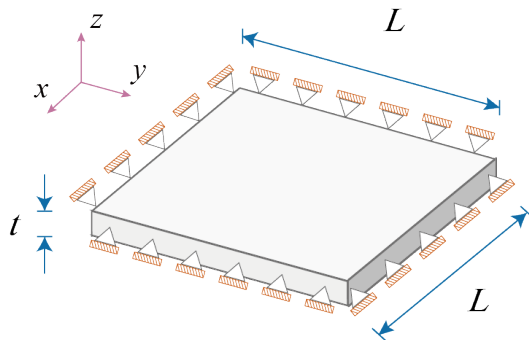
Nonlinear Modal Analysis

- Based on the updated stiffness matrix, the nonlinear eigenvalue problem is solved to extract nonlinear natural frequencies and mode shape

$$({}^t\mathbf{K}_L + {}^t\mathbf{K}_{NL} - {}^t\omega^2\mathbf{M}) {}^t\delta = 0$$

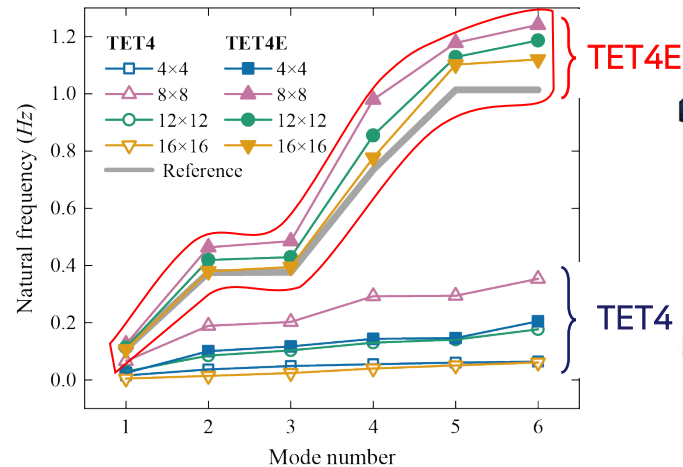
Mesh Distortion Sensitivity Analysis

Problem Description

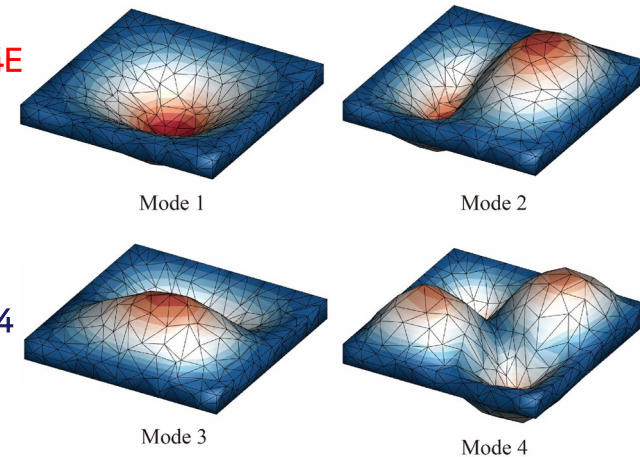


- $A_{10} = 100 \text{ kN/m}^2$
- $k = 2,000 \text{ kN/m}^2$
- $L = 10 \text{ cm}$
- $p = -0.5 \text{ N/m}^2$

Natural Frequency Comparison

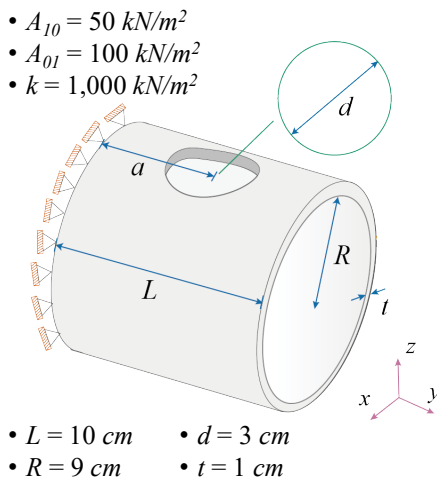


Modal Shape



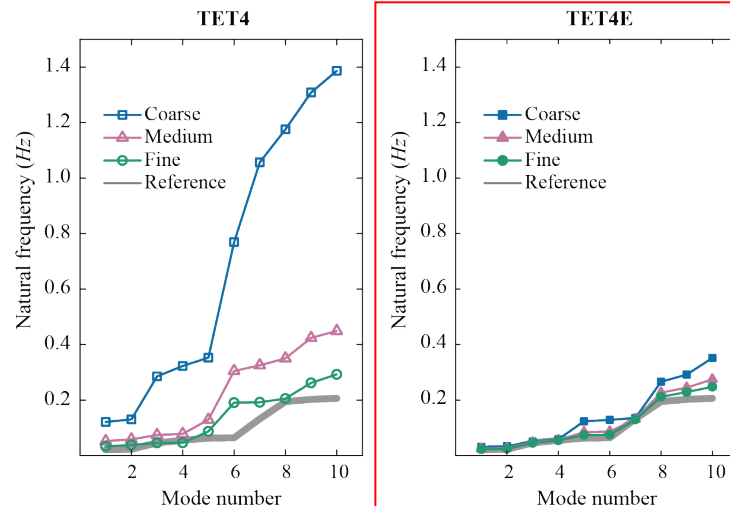
Hyperelastic Tire under Compression

Problem Description

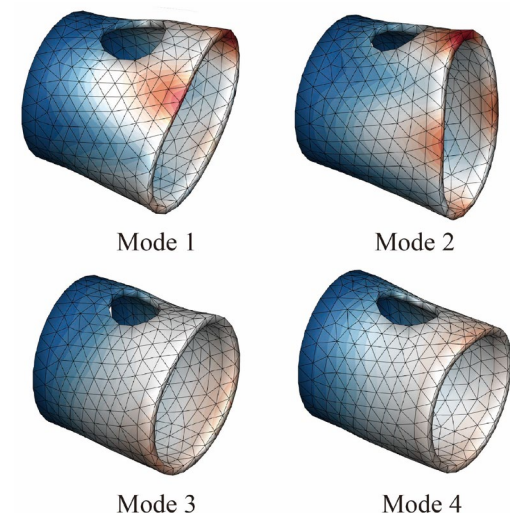


- $A_{10} = 50 \text{ kN/m}^2$
- $A_{01} = 100 \text{ kN/m}^2$
- $k = 1,000 \text{ kN/m}^2$
- $L = 10 \text{ cm}$
- $d = 3 \text{ cm}$
- $R = 9 \text{ cm}$
- $t = 1 \text{ cm}$

Vertical Displacement Curve



Modal Shape



Research Purpose and Development

- Development of PU-Based Tetrahedral Elements for Nonlinear Structural Analysis of Incompressible Hyperelastic Materials

Key Achievements

Conducted various numerical examples

- Including cantilever beams, compression blocks, cylindrical shells, and distorted plates, to verify mesh sensitivity, solution accuracy, and convergence performance

Nonlinear static analysis

- By preventing volumetric locking phenomena in incompressible materials, predicted nonlinear behavior with accuracy exceeding existing elements

Nonlinear modal analysis

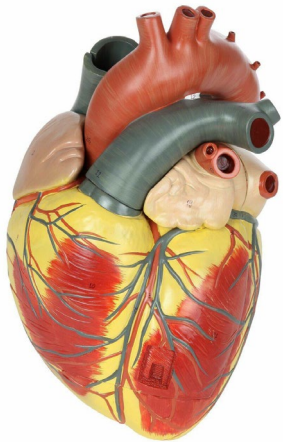
- Derived natural frequencies and mode shapes with an average accuracy higher than existing elements, even with a reduced number of elements.

Limitations

Efficiency Improvement

- Optimization is needed to address the additional computational cost introduced by the Partition of Unity approach

Virtual Heart System Modeling



Development of Simulation Tools for Virtual Heart



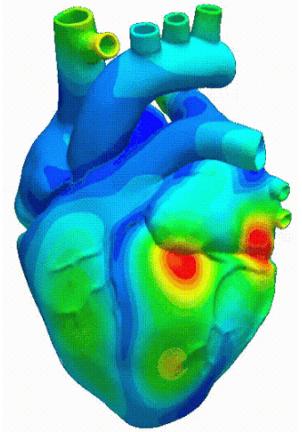
Biomaterial modeling reflecting the hyperelastic characteristics of the virtual heart



Partition of Unity-based nonlinear computational mechanics solution capable of simulating large deformations



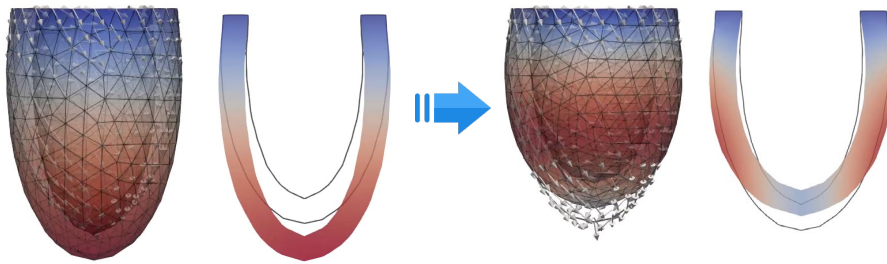
Development of algorithms reflecting cardiac characteristics (Fiber Orientation, Electric Wave Propagation)



HM Jun, 메디컬 트윈 가상심장 모델링
2021년 한국전산구조공학회

Analysis of Mechanical Properties in a Virtual Heart Using Nonlinear Modal Analysis

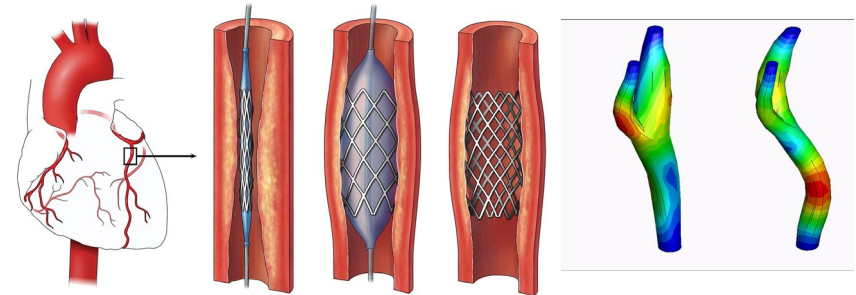
Fiber Orientation Changes



MC Trinh, HM Jun, 비선형 모드해석을 활용한 인간 심실섬유의 방향 최적화
2024년 한국전산구조공학회

Patient-specific Stent Design through Stress Concentration and Strain Analysis of the Heart

Transformation of a Deformed Form



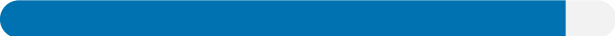
Shape design of stent

Behavior Analysis

Education

- ▶ 2023 – 2025 전북대학교 석사 (졸업예정)
- ▶ 2019 – 2023 전북대학교 학사 졸업
- ▶ 2017 – 2019 군산대학교 학사 중퇴 (편입)

Skill

- ▶ Programing Language (Fortran, Python, Matlab)

- ▶ FEA Software (ANSYS, ADINA)

- ▶ 3D CAD (SpaceClaim, FreeCad, Gmesh)

- ▶ Engineering Tool (Tecplot, Paraview, Origin ...)

- ▶ Illustrator, Word, Endnote, Exel, Power Point ...


Experience

✓ 국제활동

- ▶ 2024.12 – 2024.12 베트남 호치민, 호치민 과학대학교 및 방랑 대학교 공동 심포지엄
- ▶ 2023.01 – 2023.01 미국 네바다주 라스베이거스, 국제 가전 박람회 제품 제작 및 전시
- ▶ 2023.07 – 2023.08 미국 워싱턴 DC, 조지워싱턴 대학교 연구/테크 프론티어 프로그램

✓ 학술 연구수상

- ▶ 2021.12 – 은상, 캡스톤 경진대회
- ▶ 2022.06 – 우수발표상, 한국풍력 에너지학회
- ▶ 2022.08 – 조지워싱턴 대학교 연구/테크 프론티어
- ▶ 2023.11 – 우수발표상, 한국풍력 에너지학회
- ▶ 2023.12 – 1등, 2023 호남권 I-Corps 모의투자대회
- ▶ 2024.01 – 최우수상, 2023 기술창업기반 창업교육지원 IR 데모데이
- ▶ 2024.01 – 우수상, 과학기술정보통신부 주최 LAB START-UP 2024 IR 부문
- ▶ 2024.06 – 우수상, 2024 GIST-호남권대학 창업아이디어 경진대회

✓ 어학

- ▶ 2020.02 – TOEIC 880
- ▶ 2024.12 – Opic IM1

Publication

- ▶ 2024.12 – [Lim, T.J.](#), Trinh, M.C., Jun, H. A four-node tetrahedral element using partition of unity method for nonlinear analysis of nearly incompressible hyperelastic materials. *Will be submitted soon*

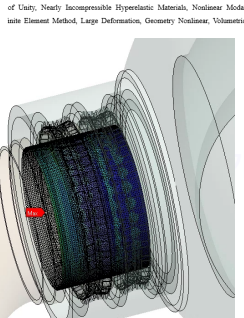
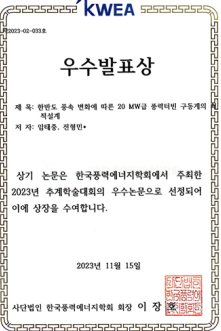
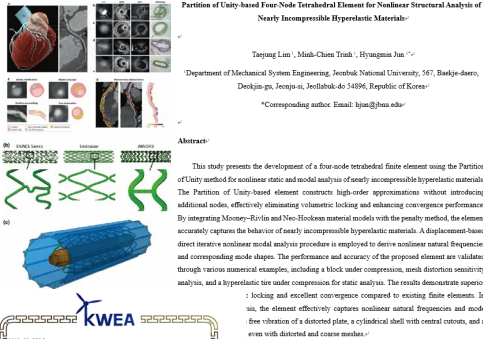
Presentations

- ▶ 2024.12 – [임태중](#), 트린민첸, 전형민. Nearly Incompressible 초탄성 재료모델을 포함하는 Partition of Unity 비선형 유한요소 개발 및 비선형 모드해석, 한국생산제조학회 2024 추계학술대회
- ▶ 2024.11 – [임태중](#), 트린민첸, 전형민. A Four-Node Tetrahedral Element Based on the Partition of Unity Method for Nonlinear Static and Modal Analysis of Nearly Incompressible Hyperelastic Materials, COSARI Joint-Conference
- ▶ 2024.04 – [임태중](#), 전형민. Partition of Unity 기반 초탄성 유한요소 개발 및 비선형 모달 분석, 한국전산구조공학회 정기학술대회
- ▶ 2023.11 – [임태중](#), 전형민. 초탄성재료 해석을 위한 PU 기반 유한요소 개발, 한국전산구조공학회 학술심포지엄
- ▶ 2023.11 – [임태중](#), 전형민. 한반도 풍속 변화에 따른 20MW급 풍력터빈 구동계 최적설계, 한국풍력에너지학회 추계학술대회
- ▶ 2023.06 – [임태중](#), 송필무, 이병효, 전형민. 한반도 풍속데이터 변화에 따른 대형풍력 메인 베어링의 피로수명 예측에 관한 연구, 한국풍력에너지학회 춘계학술대회
- ▶ 2022.06 – 송필무, 이병효, [임태중](#), 전형민. 대형 메인베어링의 기계적 성질 예측에 관한 연구, 한국풍력에너지학회 춘계학술대회

Courses

- ▶ 2024.08 - Python으로 배우보는 최신 최적설계기법의 기초와 활용(신뢰성 기반 최적설계 이론 및 실습, 구조위상최적화와 기계학습), 한국전산구조공학회, 신기술강습회
- ▶ 2023.05 – Mechanical(Basic, Motion, Dynamic, Fatigue), Fluent(heat transfer, turbulence), Ansys DesignXplorer Optimization Fundamentals, 태성에스엔이 CAE Academy
- ▶ 2022.01 – 22년도 빛가람 에너지밸리 공동캠퍼스 교육(증기터빈, 밸브, 펌프, 발전기, AVR(여자기), 송전 및 발전 시스템, 공정 제어), 전력거래소, 한전 KDN, 한전 KPS

Project



비선형 생체재료를 고려한 관상동맥 전산역학 솔루션 개발, 2024년도 건지 논문지원 프로그램 (연구책임자) & 임상데이터 및 생체신호 기반 심장혈관 생체역학 3D 모델링 시뮬레이션 SW 개발, 바이오산업기술개발사업

비선형 생체재료를 고려한 관상 동맥 전산역학 솔루션 개발

- 비선형 생체재료 특성을 반영한 관상동맥 모델링과 전산역학 시뮬레이션 SW 개발
- 생체 신호 데이터를 수집하여 3D 모델링을 수행하고, 시뮬레이션 환경을 조성
- 총 9회의 학술대회에서 발표를 진행하였으며, 1회의 SCIE 논문 발표를 예정



한반도 해역에 적합한 20 MW급 초대형 해상풍력 발전 시스템 개념설계, 사외공모 기초연구

응력 및 피로수명 해석을 통한 대규모 해상 풍력 시스템 설계

- 구조물의 피로수명 분석 절차를 설계하고 시뮬레이션을 통해 수명 예측
- 해상 조건 데이터를 체계적으로 수집하고 분석하여 설계 최적화에 반영
- 한국풍력에너지 학회에 연구 결과를 발표하여 2회의 우수 발표상을 수상



딥러닝 기반 비접촉식 가축체중측정 장치, JBNU-INNOVATION AWARDS & 딥러닝 컴퓨터비전 기반 과산계 선별 시스템, 공공기술기반 시장연계 창업탐색 지원사업, 창업진흥원 (연구책임자)

딥러닝 기반 가축의 과산계 선별 시스템의 시장 발굴 및 사업화

- 국내외 시장 인터뷰를 통해 요구사항을 파악하고, 효과적인 판매 전략을 수립
- 조지 워싱턴 대학 연구 중 최우수 팀 선정 및 목업을 제작하여 CES에 출품
- IR 발표에서 5회의 수상을 통해 프로젝트의 우수성과 시장 경쟁력을 인정받음



Hyungmin Jun
(Principal Investigator)



Minh-Chien Trinh
(Research Professor)



Taejung Lim
(B.S)

“ Thank you ! ”

Do you have any questions?

dlaxownd123@jbnu.ac.kr