Finite Element Analysis

Homework #2, Due date: 2023-05-22 (Mon.)

1. Idealize the bar structure shown as an assemblage of 2 two-node bar elements.

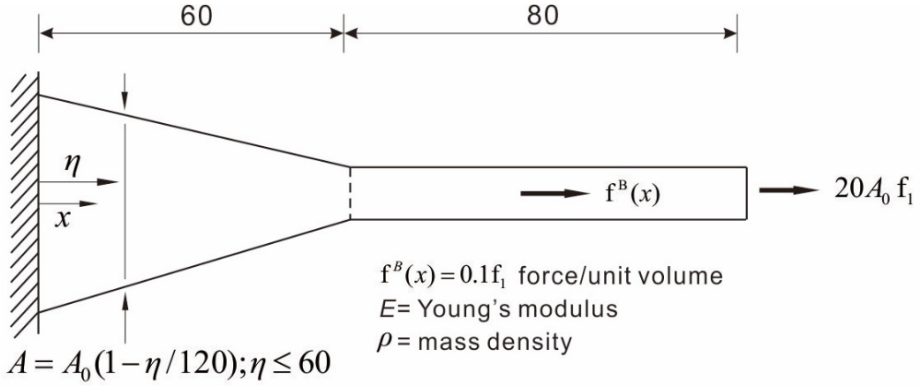

Figure 1. One-dimensional bar problem

(a) (5pt.) Calculate the equilibrium equations $\mathbf{KU} = \mathbf{R}$.

(b) (5pt.) Calculate the mass matrix of the element assemblage.

2. Consider the finite element analysis illustrated in Figure 2.

* Young's modulus E, Poisson's ratio υ, Density ρ, Gravity g, Thickness t,

Plane stress condition: $\begin{bmatrix} \tau_{xx} \\ \tau_{yy} \\ \tau_{xy} \end{bmatrix} = \frac{E}{1-\upsilon^2} \begin{bmatrix} 1 & \upsilon & 0 \\ \upsilon & 1 & 0 \\ 0 & 0 & \frac{1-\upsilon}{2} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix}$ with $\upsilon = 0$.

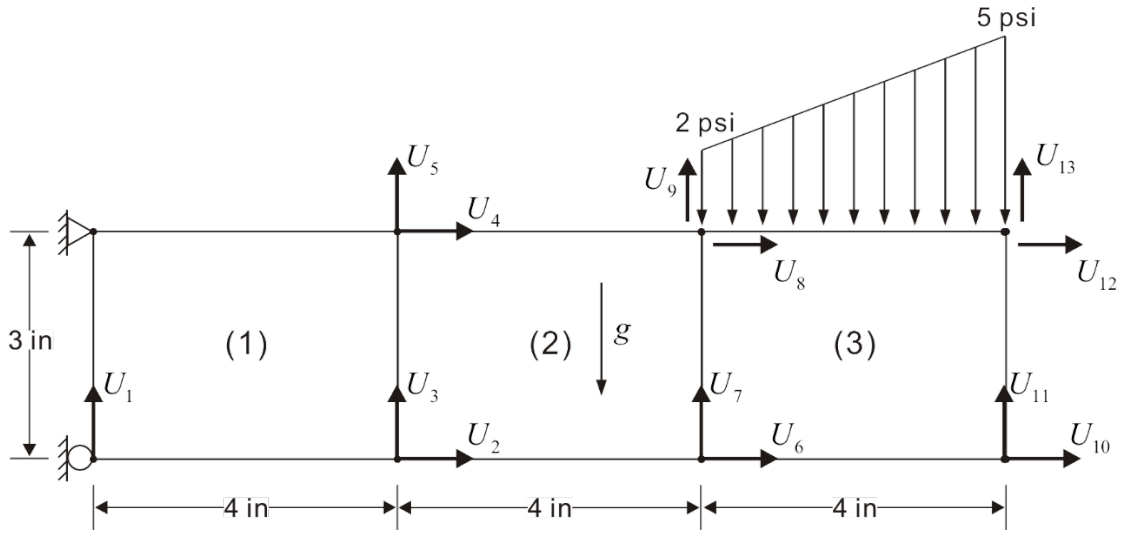

Figure 2. Cantilever beam problem

(a) (5pt.) Referring to Figure 3, establish the **H** and **B** matrices of an element, in which the nodal DOF vector for the element is defined by $\hat{\mathbf{u}} = [u_1\ u_2\ u_3\ u_4\ v_1\ v_2\ v_3\ v_4]^T$.

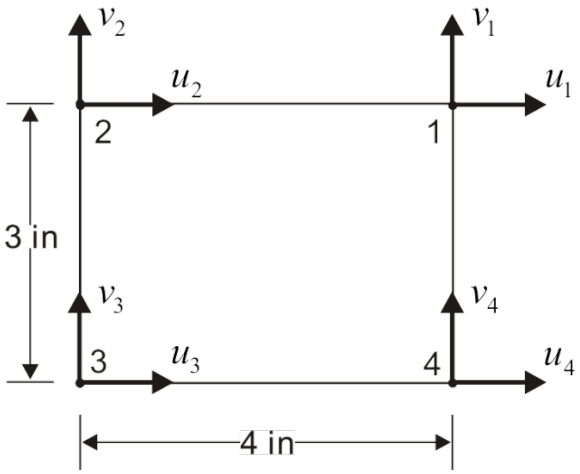

Figure 3. The 4-node element

(b) (10pt.) Calculate the components of the **K** matrix, $K_{U_2U_2}$, $K_{U_6U_7}$, $K_{U_7U_6}$ and $K_{U_5U_{12}}$ of the structural assemblage.

(c) (5pt.) Calculate the nodal load R_9 due to the body force and linearly varying surface force.