유한요소해석

Homework #3

1. Consider the isoparametric 4-node plane stress element shown in Fig. 1.

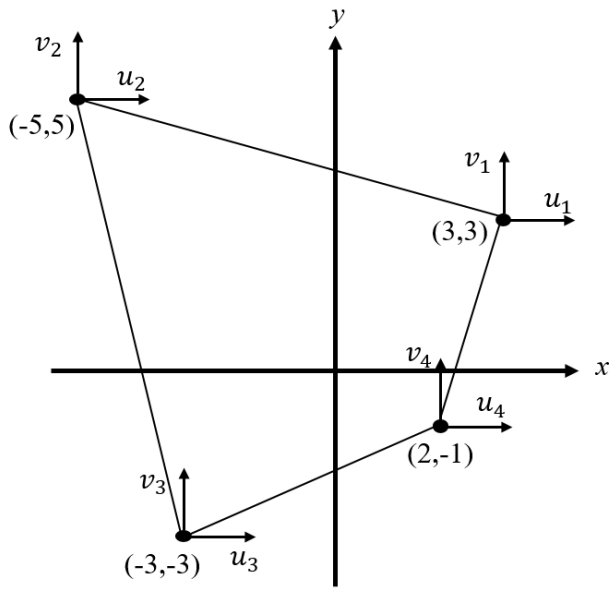

Fig. 1. A 4-node plane stress element

(a) Calculate the Jacobian matrix $\mathbf{J}(r,s)$ at $r = s = 0$.

(b) Give the strain-displacement matrix $\mathbf{B}(r,s)$ at $r = s = 0$. Note that the nodal displacement vector is $\mathbf{U} = \begin{bmatrix} u_1 & u_2 & u_3 & u_4 & v_1 & v_2 & v_3 & v_4 \end{bmatrix}^T$.

(c) Calculate the component of the stiffness matrix corresponding to u_1 and v_1, $K_{u_1 v_1}$ using one-point Gaussian integration.

Note that thickness = 1.0 and $\mathbf{C} = \begin{bmatrix} E & 0 & 0 \\ 0 & E & 0 \\ 0 & 0 & E/2 \end{bmatrix}$.

* In the one-point Gaussian integration, Gauss point is $r = s = 0$ and weight factor is 2.

2. We consider a triangular cantilever problem modeled by a 3-node plane stress element as shown Fig. 2. The displacement BCs are presented in Fig. 2, thickness is 1.0,

$$\boldsymbol{\varepsilon} = \begin{Bmatrix} \varepsilon_{XX} \\ \varepsilon_{YY} \\ \gamma_{XY} \end{Bmatrix} \quad \text{and} \quad \mathbf{C} = \frac{E}{(1-\upsilon^2)} \begin{bmatrix} 1 & \upsilon & 0 \\ \upsilon & 1 & 0 \\ 0 & 0 & (1-\upsilon)/2 \end{bmatrix} \text{ with } \upsilon = 0 .$$

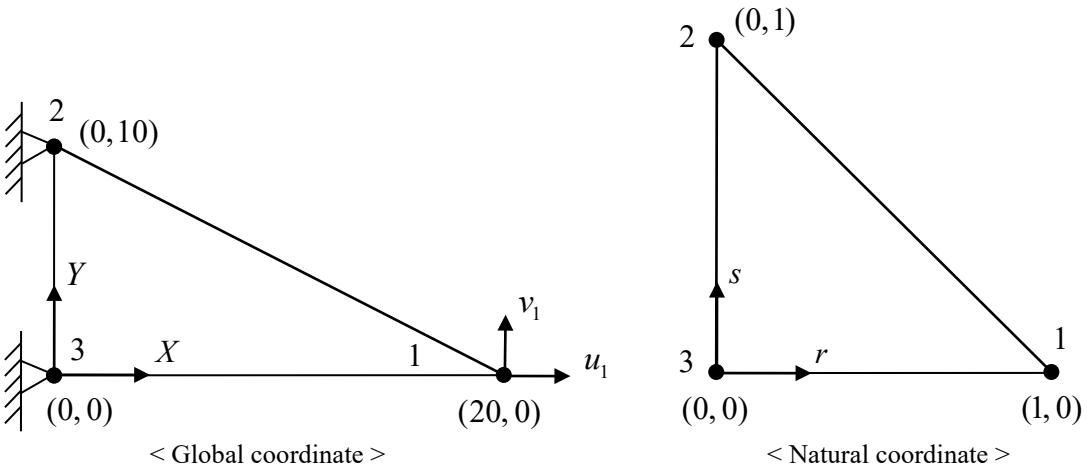

Fig. 2. A triangular cantilever

The shape functions of the 3-node element are given by

$$h_1 = r , \quad h_2 = s \quad \text{and} \quad h_3 = 1 - r - s .$$

Calculate the followings

(a) Jacobian matrix $\mathbf{J}(r,s)$

(b) Strain-displacement matrix $\mathbf{B}(r,s)$ corresponding to global nodal displacement vector $\mathbf{U} = \begin{bmatrix} u_1 & v_1 \end{bmatrix}^T$

(c) Stiffness matrix $\mathbf{K}$ [2x2] corresponding to global nodal displacement vector